Name: _____   RED ID: _____

Fall 2023 Math 245 Exam 1

Please read the following directions:

Please write legibly, with plenty of white space. Please **print** your name and REDID in the designated spaces above, and your initials on the second page. Please fit your answers into the designated areas; material outside the designated areas (such as on this cover page) will not be graded. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. The first two problems are worth 5-11 points while the remaining problems are worth 8-16 points, for a minimum score of 74/150 and a maximum score of 150/150. The use of notes, books, calculators, or other materials on this exam is strictly prohibited, except you may bring one 3"x5" card (both sides) with your handwritten notes. This exam will begin at 10:00 and will end at 10:50; pace yourself accordingly. Good luck!

Special exam instructions for HH-130:
1. Please stow all bags/backpacks/boards at the front of the room. All contraband, except phones, must be stowed in your bag. All smartwatches and phones must be silent, non-vibrating, and either in your pocket or stowed in your bag.
2. Please remain quiet to ensure a good test environment for others.
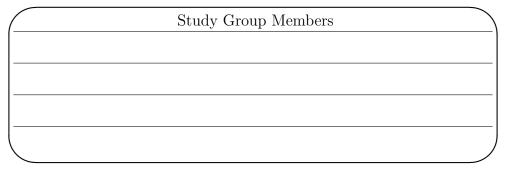3. Please keep your exam on your desk; do not lift it up for a better look.
4. If you have a question or need to use the restroom, please come to the front. Bring your exam. I cannot come to you unless you are sitting by an aisle.
5. If you are done and want to submit your exam and leave, please wait until one of the three designated exit times, listed below. Please do **NOT** leave at any other time. If you are sure you are done, just sit and wait until the next exit time, with this cover sheet visible.

Designated exam exit times:
   10:20 "I need to work harder"
   10:40 "I can't wait to get out of here"
   10:50 "I need every second I can get"

In the box below, please print the full names of everyone in your regular study group (no need to list yourself). If everyone in your study group agrees on who's in it, each person will receive +1 extra credit on this exam.

Study Group Members

REMINDER: Use complete sentences.

Problem 1. Carefully define the following terms:

   a. composite

   b. converse

Problem 2. Carefully state the following theorems:

   a. Division Algorithm Theorem

   b. Disjunctive Syllogism Theorem

Problem 3. Use a truth table to help prove the following:
For all propositions p, q, we have $(p \uparrow q), (p \rightarrow q) \vdash \neg p$.

Problem 4. Let p, q be propositions. Without using truth tables, prove $p \wedge q \equiv q \wedge p$.
Note: do not use/cite commutativity of $\wedge$ – you are being asked to prove commutativity!

Problem 5. Let p, q be propositions. Use semantic theorems to prove the "Trivial Proof Theorem": $q \vdash p \rightarrow q$. Do not use the theorem to prove itself!

Problem 6. Prove or disprove: For all $a, b, c \in \mathbb{Z}$, if $ac|b$ then $a|b$.

Problem 7. Prove or disprove: For all $x \in \mathbb{Z}$, $|4x + 9| > 1$.

Problem 8. Let $a, b \in \mathbb{N}_0$. Use the definition of $\leq$ to prove that if $2a \leq b$ then $2a^2 + ab \leq b^2$.

Problem 9. Prove or disprove: For all $a, b, c \in \mathbb{Z}$, if $a|b$, then $ac^2|b^2c$.

Problem 10. Prove or disprove: $\forall x, y \in \mathbb{R}, (x < y) \to (\exists z \in \mathbb{R}, x < z < y)$.